

Summary of

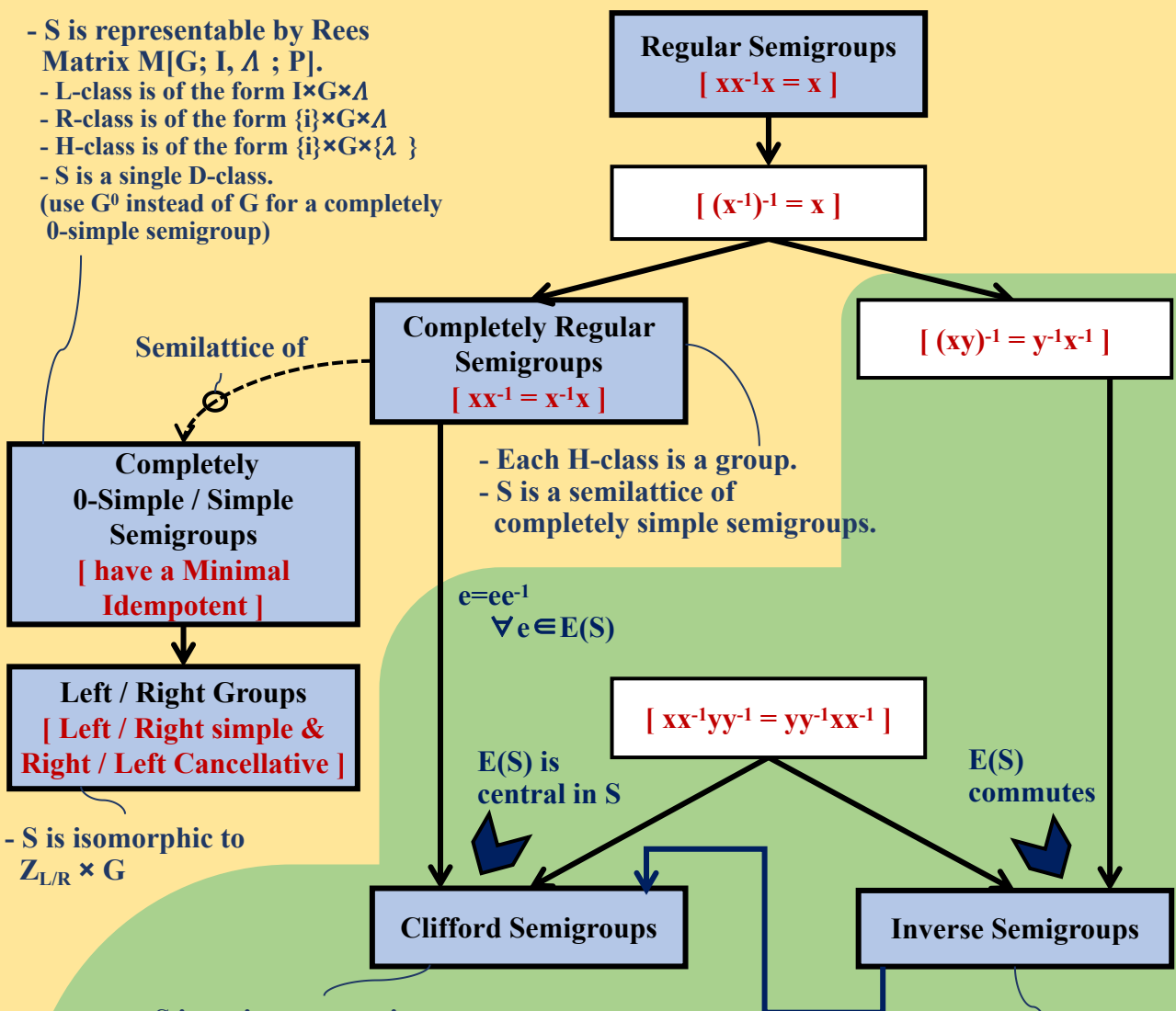
“Chapter 4 Regular semigroups” and “Chapter 5 Inverse semigroups” in
Alan J. Cain : Nine Chapters on the Semigroup Art.
 Lecture notes for a tour through semigroups

résumé by Akihiko Koga
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Chapter 4 Regular Semigroups

Below, S is a semigroup that satisfies given conditions.

- S is representable by Rees Matrix $M[G; I, \Lambda; P]$.
- L-class is of the form $I \times G \times \Lambda$
- R-class is of the form $\{i\} \times G \times \Lambda$
- H-class is of the form $\{i\} \times G \times \{\lambda\}$
- S is a single D-class.
 (use G^0 instead of G for a completely 0-simple semigroup)



- S is isomorphic to $Z_{L/R} \times G$

- Each H-class is a group.
- S is a semilattice of completely simple semigroups.

$$e = ee^{-1} \quad \forall e \in E(S)$$

- S is an inverse semigroup.
- Each D-class has a unique idempotent.
- S is isomorphic to Strong Semilattice of Groups.

- This is equivalent to regular & $E(S)$ commutes
- S is embedable into the semigroup of partial injective functions I_X .
 (Vagner-Preston Representation Theorem)
- $\forall x \in S$ has unique inverse.
- Each L/R-class has just one idempotent.
- S has the natural order with $x \leq y$ iff $x = xx^{-1}y$
- Free inverse semigroup $FInvS(A)$ has a decidable method to check the given equation using the Munn tree.

Chapter 5 Inverse Semigroups