

# Main Topics of

“Chapter4 Regular semigroups” and “Chapter5 Inverse semigroups” in  
**Alan J. Cain : Nine Chapters on the Semigroup Art.**  
 Lecture notes for a tour through semigroups

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## Chapter 4 Regular Semigroups

Regular Semigroups  
 $[ xx^{-1}x = x ]$

$[ (x^{-1})^{-1} = x ]$

representable by  
 Rees Matrix

Semilattice of

Completely Regular  
 Semigroups  
 $[ xx^{-1} = x^{-1}x ]$

$[ (xy)^{-1} = y^{-1}x^{-1} ]$

Completely  
 0-Simple / Simple  
 Semigroups  
 $[ \text{have a Minimal Idempotent} ]$

Left / Right Groups  
 $[ \text{Left / Right simple \& Right / Left Cancellative} ]$

isomorphic to  
 $Z_{L/R} \times G$

$[ xx^{-1}yy^{-1} = yy^{-1}xx^{-1} ]$  \*

Clifford Semigroups

Inverse Semigroups

isomorphic to Strong  
 Semilattice of Groups

embedable into the semigroup of  
 partial injective functions  $I_X$   
 (Vagner-Preston Representation  
 Theorem)

## Chapter 5 Inverse Semigroups

\*) The equation  $xx^{-1}yy^{-1} = yy^{-1}xx^{-1}$  seems slightly complex and hard to grasp its meaning. Note that  $xx^{-1}$  is an idempotent since  $(xx^{-1})(xx^{-1}) = (xx^{-1}x)x^{-1} = xx^{-1}$ . Therefore, the equation shows that idempotents of the form  $xx^{-1}$  commute each other, and depending on other conditions, it lead to the commutabilites of  $E(S)$  of several degrees. Commutative  $E(S)$  plays an impotant role in chapter 4 and chapter 5.